# Harmony and Symmetry in our Solar System

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# Harmony-----> Math <-----Astronomy

The Quadrivium in Plato's **Republic** (c 375 BC)

= Arithmetic, Geometry, Astronomy, and Harmony

Plato's **Timaeus** (c 360 BC)

“(H)e proceeded to divide the entire mass into portions related to one another
by double ratios -1, 2, 4, 8- and triple -1, 3, 9, 27-,
and then to fill up these intervals using two kinds of means.
The first mean exceeds and is exceeded by equal parts of its extremes;
the second kind of mean exceeds and is exceeded by an equal number.

These portions were divided by him lengthwise, which he united at the center like the letter X,
and bent the arms into a circle or sphere.

Seven unequal orbits were distributed: three of them, the Sun, Mercury, Venus, with equal swiftness, and the remaining four, the Moon, Saturn, Mars, Jupiter, with unequal swiftness to the three and to one another, but all in due proportion.”

# Deriving Musical Ratios (Pythagoras c 500 BC)

- Fundamental Ratio of string lengths is 2 to 1, **2/1 = “Octave”**
- The octave can be divided by taking two different means
- (For a = 1 and b = 2)
- **Arithmetic Mean** = (a + b) / 2, **3/2 = “Perfect Fifth”**
- **Harmonic Mean** = 2ab / (a + b) **4/3 = “Perfect Fourth”**

# In the 16th Century, musical theorists such as Zarlino completed the set of Harmonic Consonances *

- Adding the four more intervals: the **Minor Third,** the **Major Third,** the **Minor Sixth,** and the **Major Sixth**
- To the Pythagorean Set of four: **Unison**, **Octave**, **Perfect Fourth**, and **Perfect Fifth**

- (*Musical Ratios that sound pleasant when played together)
- https://www.youtube.com/watch?v=gYtSI4-ShLU

# Musical Ratios

| **Musical Ratio** (frequency or string length) | **Interval** |
|---|---|
| 1/1 = 1 | Unison |
| 6/5 = 1.20 | Minor Third |
| 5/4 = 1.25 | Major Third |
| 4/3 = 1.33 | Perfect Fourth |
| 3/2 = 1.5 | Perfect Fifth |
| 8/5 = 1.6 | Minor Sixth |
| 5/3 = 1.67 | Major Sixth |
| 2/1 = 2 | Octave |

# Johannes Kepler
# Harmonices Mundi (1619) Book 5, Chapter 4

IN WHAT THINGS HAVING TO DO WITH THE PLANETARY MOVEMENTS HAVE THE HARMONIC CONSONANCES BEEN EXPRESSED BY THE CREATOR, AND IN WHAT WAY?

# Planetary Measurements

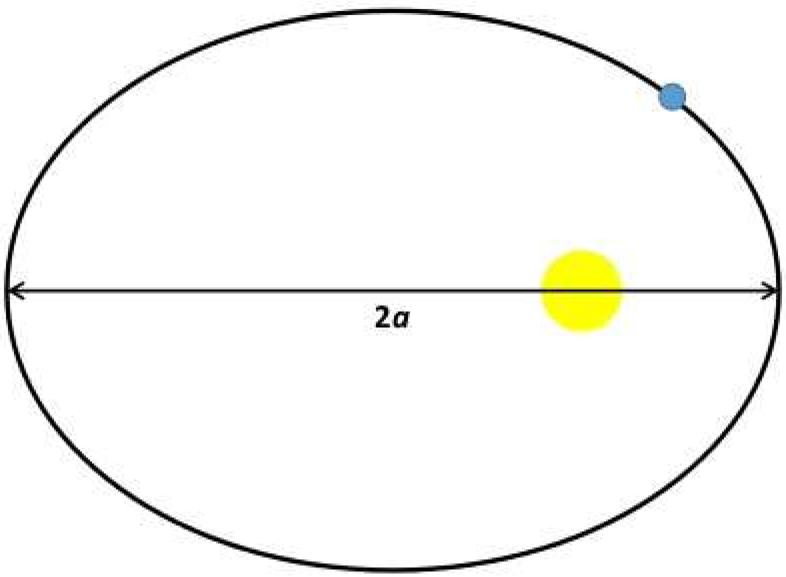
- 1) Distances from the Sun
- 2) Periodic Times
- 3) Velocities

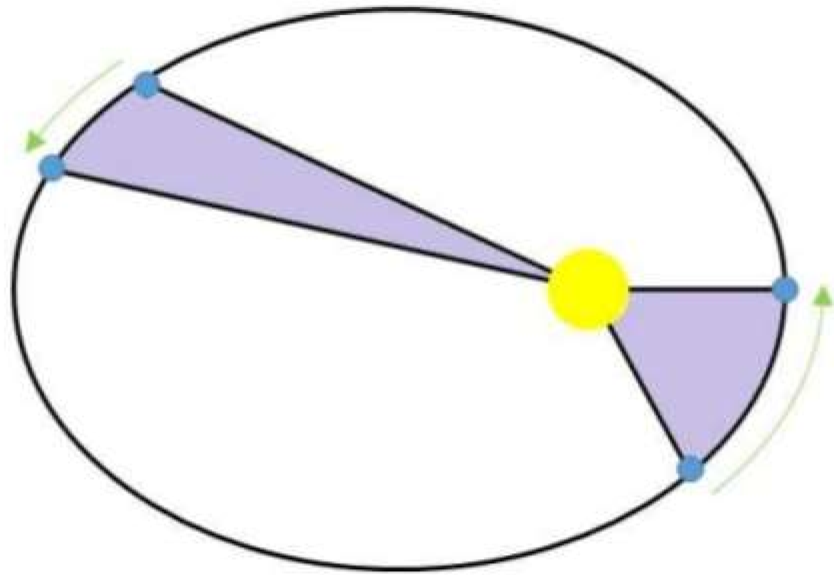
(**Dynamic** versus **Static** as seen in Kepler's laws)

# Kepler's laws of planetary motion

1. The orbit of a planet is an ellipse with the Sun at its focus.
2. The line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.
3. The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

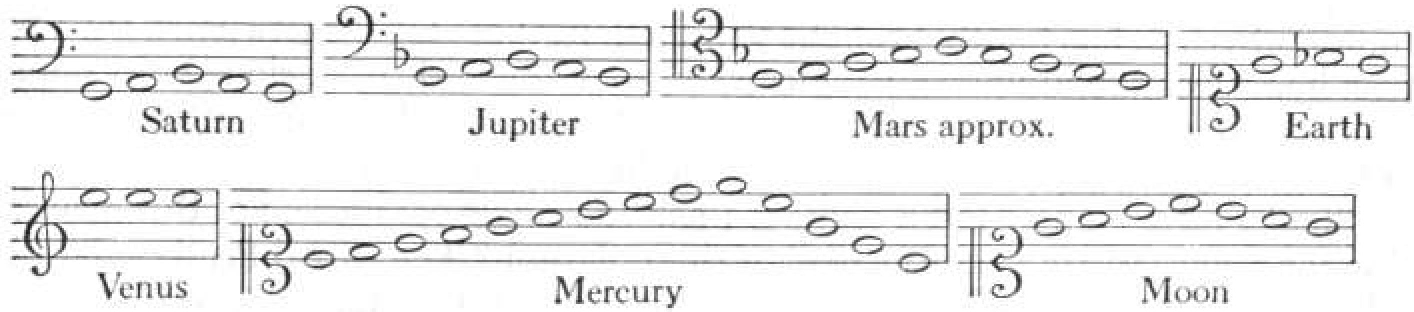
$$\tau^2 \propto a^3$$



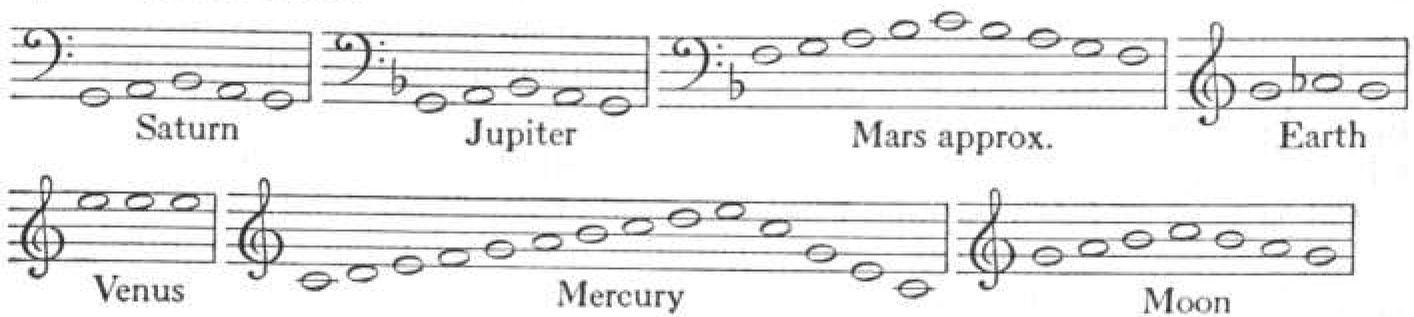



| Of Two Planets | | | | | Of Single Planets |
|---|---|---|---|---|---|
| *Converging* | *Diverging* | | | | |
| | | Saturn's aphelion | 10,052. | a. | More than a minor whole tone $\frac{10,000}{9,000}$ |
| $\frac{a}{d}=\frac{2}{1}$, | $\frac{b}{c}=\frac{5}{3}$ | perihelion | 8,968. | b. | Less than a major whole tone $\frac{10,000}{8,935}$ |
| | | Jupiter's aphelion | 5,451. | c. | No concordant ratio but approximately 11 : 10, a discordant or diminished 6 : 5. |
| | | perihelion | 4,949. | d. | |
| $\frac{c}{f}=\frac{4}{1}$, | $\frac{d}{e}=\frac{3}{1}$ | | | | |
| | | Mar's aphelion | 1,665. | e. | Here 1662 : 1385 would be the consonance 6 : 5, and 1665 : 1332 would be 5 : 4 |
| | | perihelion | 1,382. | f. | |
| $\frac{e}{h}=\frac{5}{3}$, | $\frac{f}{g}=\frac{17}{20}$ | | | | |
| | | Earth's aphelion | 1,018. | g. | Here 1025 : 984 would be the diesis 24 : 25. Therefore it does not have the diesis. |
| | | perihelion | 982. | h. | |
| $\frac{g}{k}=\frac{2}{1\frac{1}{2}}$ viz. $\frac{1000}{710}$, | $\frac{h}{i}=\frac{27}{20}$ | | | | |
| | | Venus' aphelion | 729. | i. | Less than a sesquicomma. |
| | | perihelion | 719. | k. | More than one third of a diesis. |
| $\frac{i}{m}=\frac{12}{5}$, | $\frac{k}{i}=\frac{243}{160}$ | | | | |
| | | Mercury's aphelion | 470. | l. | 243 : 160, greater than a perfect fifth but less than a harmonic 8 : 5 |
| | | perihelion | 307. | m. | |

| Harmonies Between Two Planets: *Diverging* | Harmonies Between Two Planets: *Converging* | Apparent Diurnal Movements | | Harmonies Between the Movements of Single Planets |
|---|---|---|---|---|
| | | Saturn at aphelion | 1′46″ a. | 1 : 48″ : 2′15″ = 4 : 5, |
| | | at perihelion | 2′15″ b. | major third |
| $\frac{a}{d}=\frac{1}{3}$, | $\frac{b}{c}=\frac{1}{2}$ | | | |
| | | Jupiter at aphelion | 4′30″ c. | 4′35″ : 5′30″ = 5 : 6, |
| | | at perihelion | 5′30″ d. | minor third |
| $\frac{c}{f}=\frac{1}{8}$, | $\frac{d}{e}=\frac{5}{24}$ | | | |
| | | Mars at aphelion | 26′14″ e. | 25′21″ : 38′1″ = 2 : 3, |
| | | at perihelion | 38′1″ f. | the fifth |
| $\frac{e}{h}=\frac{5}{12}$, | $\frac{f}{g}=\frac{2}{3}$ | | | |
| | | Earth at aphelion | 57′3″ g. | 57′28″ : 61′18″ = 15 : 16, |
| | | at perihelion | 61′18″ h. | semitone |
| $\frac{g}{k}=\frac{3}{5}$, | $\frac{h}{i}=\frac{5}{8}$ | | | |
| | | Venus at aphelion | 94′50″ i. | 94′50″ : 98′47″ = 24 : 25, |
| | | at perihelion | 97′37″ k. | diesis |
| $\frac{i}{m}=\frac{1}{4}$, | $\frac{k}{l}=\frac{3}{5}$ | | | |
| | | Mercury at aphelion | 164′0″ l. | 164′0″ : 394′0″ = 5 : 12, |
| | | at perihelion | 384′0″ m. | octave and minor third |




[In Modern notation:




—E. C. Jr.]

# Kepler's conception of the “Music of the Worlds” reflected the polyphony of his day

e.g.

Palestrina Missa Brevis (circa 1558)

https://www.youtube.com/watch?v=Ot6Cv8T3pAs

Kepler emphasized examination of DYNAMIC quantities in his *Harmonies of the World,* but neglected the in depth analysis of STATIC quantities. We will examine these..

- https://nssdc.gsfc.nasa.gov/planetary/factsheet/planet_table_ratio.html

- Static Quantities - Average Distance, Orbital Period, and Average Velocity

- For Planets, **m** = Mercury, **V** = Venus, **E** = Earth, **M** = Mars, (**C** = Ceres)*, **J**=Jupiter, **S** = Saturn, **U** = Uranus, and **N** = Neptune

# Musical Ratios in
# **Average Distance** Ratios of Adjacent Planets

| Planets | Ratio | Musical Ratio |
|---|---|---|
| V/m | 1.868 | No |
| E/V | 1.383 | No |
| M/E | 1.524 | Approx **3/2** |
| C/M | 1.82 | No |
| J/C | 1.88 | No |
| S/J | 1.842 | No |
| U/S | 2.004 | **2/1** |
| N/U | 1.565 | No |

# Musical Ratios in
# **Orbital Period** Ratios of Adjacent Planets

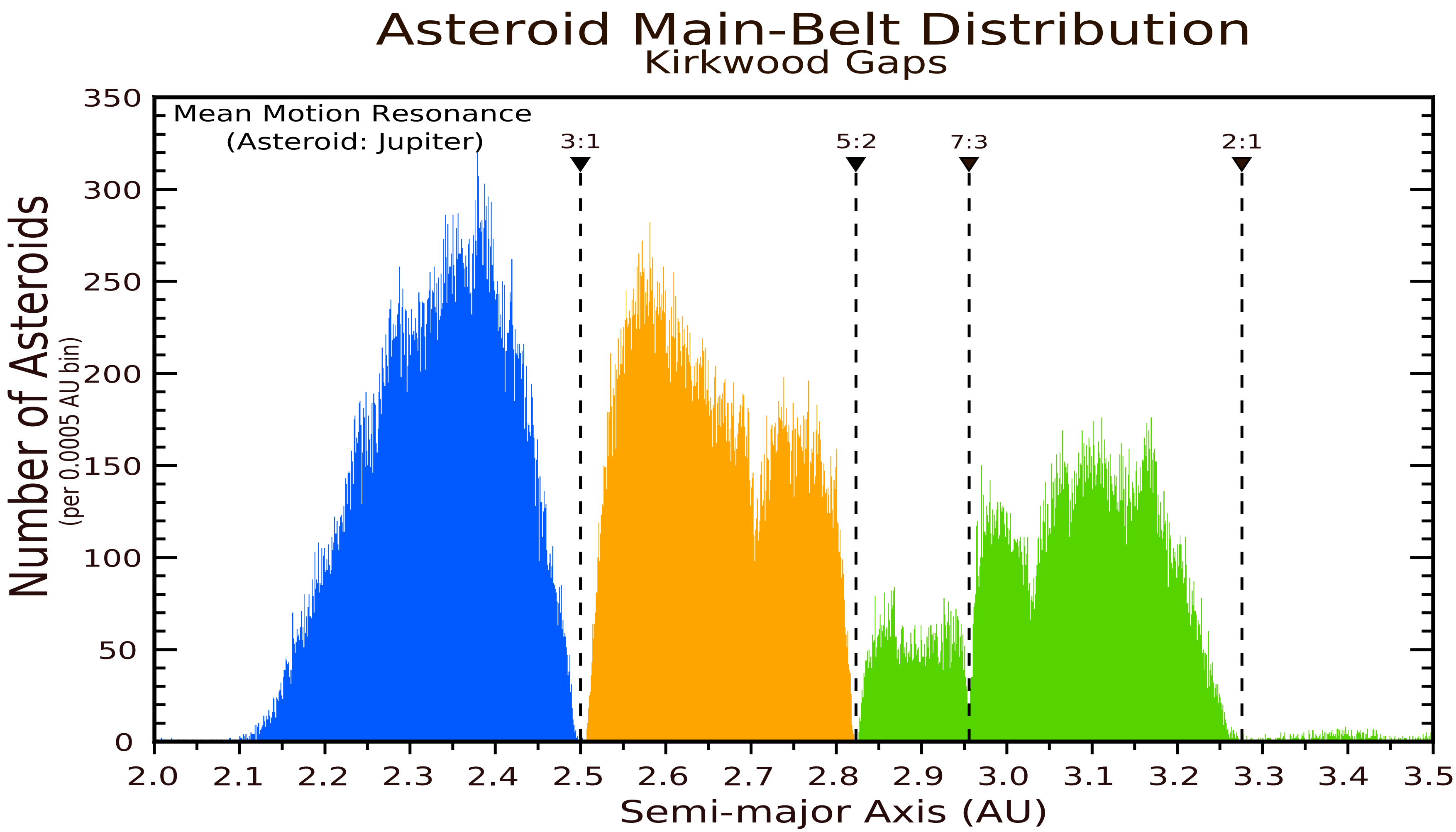
| Planets | Ratio | ( x Power of 2) | Simple Ratio | Musical Ratio |
|---|---|---|---|---|
| V/m | 2.553 | -1 | 1.277 | Approx **5/4** |
| E/V | 1.625 | 0 | 1.625 | Approx **8/5** |
| M/E | 1.881 | 0 | 1.881 | No |
| C/M | 2.45 | -1 | 1.26 | Approx **5/4** |
| J/C | 2.57 | -1 | 1.26 | Approx **5/4** |
| S/J | 2.48 | -1 | 1.24 | **5/4** |
| U/S | 2.846 | -1 | 1.42 | No |
| N/U | 1.955 | 0 | 1.955 | No |

# Orbital Resonance Phenomena can express Musical Ratios

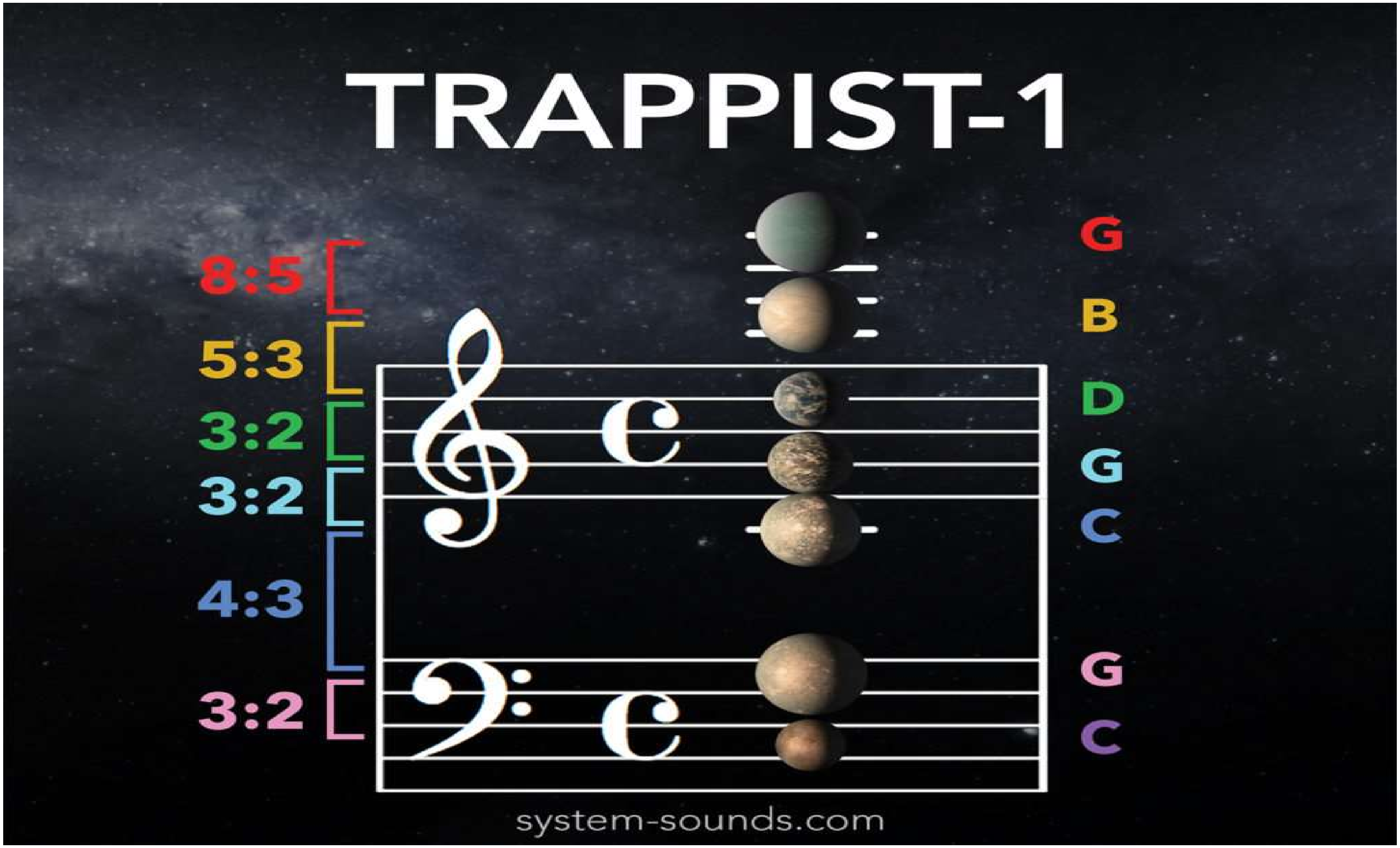
- Galilean Moons of Jupiter
  https://en.wikipedia.org/wiki/Orbital_resonance#/media/File:Galilean_moon_Laplace_resonance_animation_2.gif

- Kirkwood Gaps

- Trappist-1 System



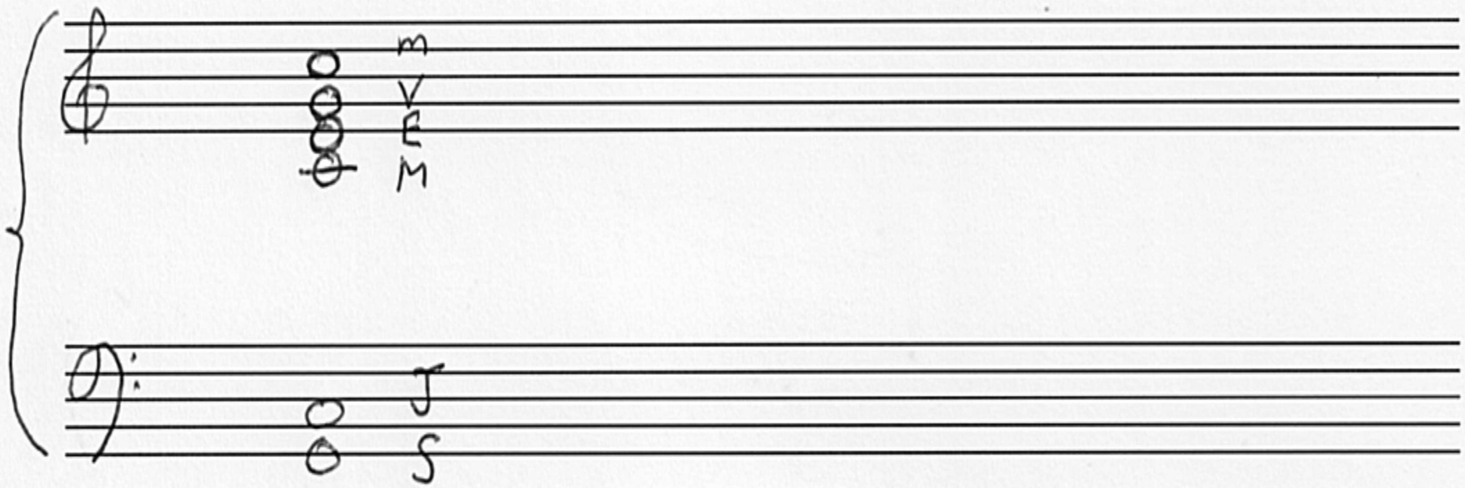
Asteroid Main-Belt Distribution
Kirkwood Gaps
Mean Motion Resonance
(Asteroid: Jupiter)
3:1
5:2
7:3
2:1
Number of Asteroids
(per 0.0005 AU bin)
0
50
100
150
200
250
300
350
2.0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3.0
3.1
3.2
3.3
3.4
3.5
Semi-major Axis (AU)


TRAPPIST-1
8:5
5:3
3:2
3:2
4:3
3:2
G
B
D
G
C
G
C
system-sounds.com

# Musical Ratios in
# **Average Velocity** Ratios of Adjacent Planets

| Planets | Ratio | Musical Ratio |
|---|---|---|
| m/V | 1.35 | **4/3** |
| V/E | 1.18 | Approx **6/5** |
| E/M | 1.24 | **5/4** |
| M/C | 1.35 | **4/3** |
| -------------------(M/J) | ---------------(1.84) | No |
| C/J | 1.37 | Approx **4/3** |
| J/S | 1.35 | **4/3** |
| S/U | 1.43 | No |
| U/N | 1.25 | **5/4** |

# Kepler's Planetary Chord (1599)



# Musical Ratios in the Ratios of Planetary Measurements

- 1) Average Distance, ***a*** (2 of 8 correct)
- 2) Periodic Times, ***t*** (5 of 8 correct)
- 3) Average Velocities, ***v*** (7 of 8 correct)

- Is this the best we can do?

- Relations between the Ratios of Planetary Measurements

$$(a_1/a_2)^{3/2} = (t_1/t_2)$$

Kepler's Third Law

$$(a_1/a_2)^{-1/2} = (v_1/v_2)$$

(By approximating the orbital perimeter as $2\Pi a$,
and using definition of $v = 2\Pi a/t$)

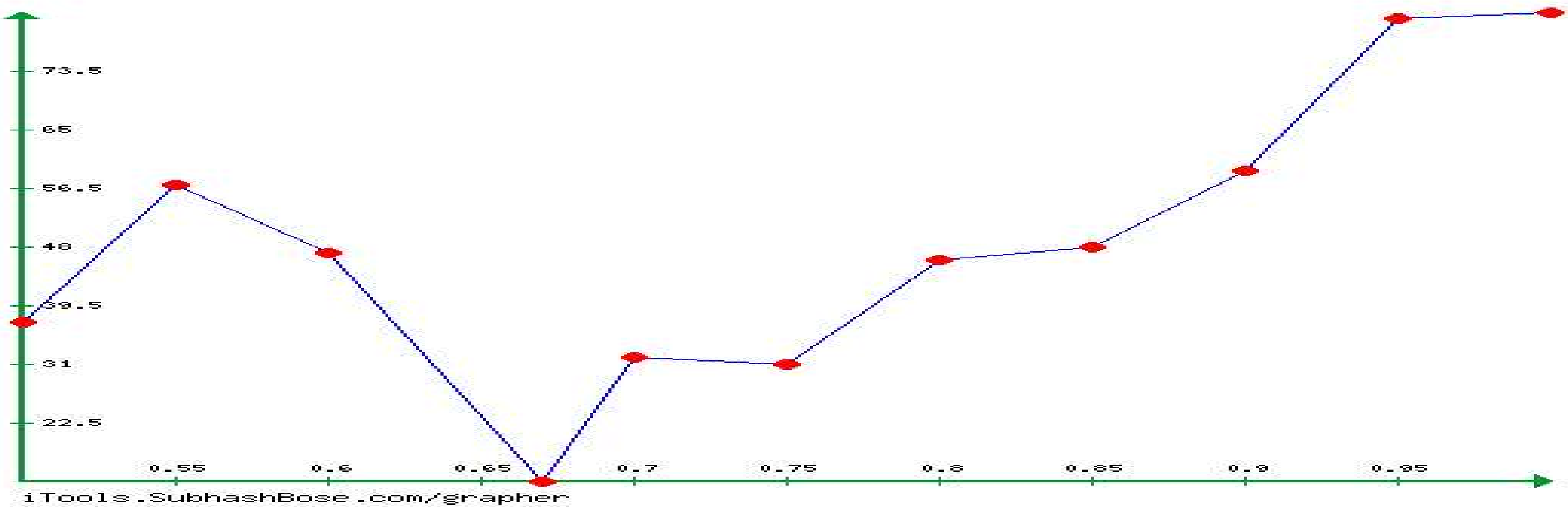
Ratios of **Average Distances, Orbital Periods, and Average Velocities** are related by *varying the* ***exponents*** from 1, to $^3/_2$, to -½ respectively.

- We have looked at three *discrete* exponents. Is there an **exponent** of planetary distance ratios that best expresses the ratios of simple harmonic consonances?

- I calculated how well musical ratios were expressed for exponents *continuously* ranging from ½ to 1

How well musical ratios were expressed for exponent ranging from ½ to 1 (for ratios of Average Distances)

Exponent vs Average Error (% Half-step) from Musical Ratios



# Musical Ratios are best expressed when distance ratios are raised to the power of 2/3 (with $p<0.001$)

| Planets | $(a_1/a_2)$ | $(a_1/a_2)^{2/3}$ | Musical Ratios |
|---|---|---|---|
| V/m | 1.868 | 1.52 | **3/2 Perfect Fifth** |
| E/V | 1.383 | 1.24 | **5/4 Major Third** |
| M/E | 1.524 | 1.32 | **4/3 Perfect Fourth** |
| C/M | 1.82 | 1.49 | **3/2 Perfect Fifth** |
| J/C | 1.87 | 1.52 | **3/2 Perfect Fifth** |
| S/J | 1.842 | 1.50 | **3/2 Perfect Fifth** |
| U/S | 2.00 | 1.59 | **8/5 Minor Sixth** |
| N/U | 1.565 | 1.35 | **4/3 Perfect Fourth** |

# Finding Symmetry

**Ceres**

3/2 3/2

**Mars** (2.25) **Jupiter**

4/3 3/2

**Earth** (4.5) **Saturn**

5/4 8/5

**Venus** (9) **Uranus**

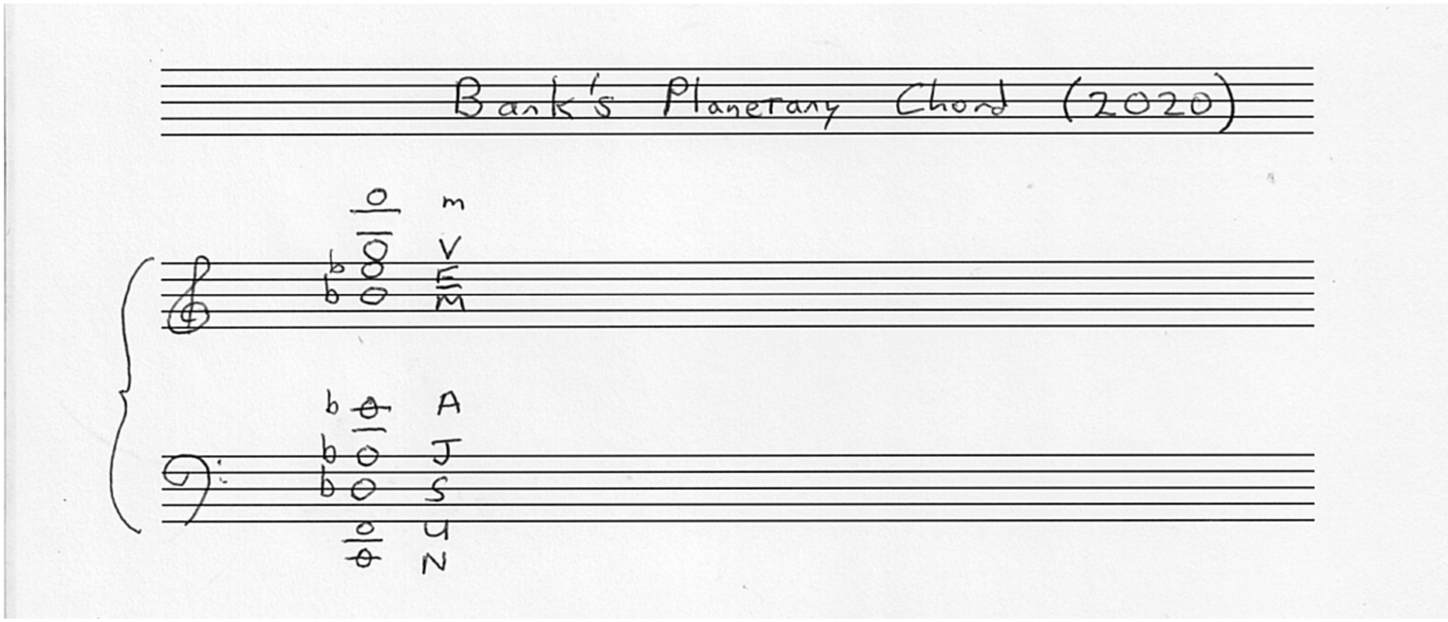
3/2 4/3

**Mercury** (18) **Neptune**

# Mirror Pairing of Planets around Asteroid Belt

| Planets | $(a_1/a_2)^{2/3}$ | $(a_1/a_2)$ Predicted | $(a_1/a_2)$ Observed |
|---|---|---|---|
| N/m | 18 | 76.37 | 77.65 |
| U/V | 9 | 27 | 26.56 |
| S/E | 4.5 | 9.546 | 9.58 |
| J/M | 2.25 | 3.375 | 3.41 |

# Planetary Chord for $(a_1/a_2)^{2/3}$



# Conclusions

- 1) Harmony - The musical ratios are best expressed by raising the average distance ratios of adjacent planets to the 2/3 power.
- 2) Symmetry - Pairing planets around the asteroid belt in a mirrored fashion, the the 2/3 power of the distance ratios double from one pair to the next (that is from J/M to S/E to U/V to N/m)
- Questions – is there a theoretical or other observational support for these findings?
- THANK YOU!!!